Improving U.S. stock return forecasts: A “fair-value” CAPE approach

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ABSTRACT

The accuracy of U.S. stock return forecasts based on the cyclically-adjusted P/E (CAPE) ratio has deteriorated since 1985. The issue is not the CAPE ratio, but CAPE regressions that assume it reverts mechanically to its long-run average. Our approach conditions mean reversion in the CAPE ratio on real (not nominal) bond yields, reducing out-of-sample forecast errors by as much as 50%. At present, low real bond yields imply low real earnings yields and an above-average “fair-value” CAPE ratio. Nevertheless, with Shiller’s CAPE ratio now well above its fair value, our model predicts muted U.S. stock returns over the next decade.

Keywords: Stock returns, return predictability, CAPE ratio, real bond yields

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Professors John Campbell and Robert Shiller's (1988) cyclically-adjusted P/E (or, CAPE) ratio is arguably the most widely-followed metric in the investment profession to judge whether or not a stock market is fairly valued. The CAPE ratio's popularity is due in part to the power of mean reversion. A high (low) cyclically-adjusted P/E (CAPE) ratio has been associated with below-average (above-average) 10-year-ahead U.S. stock returns.

Nevertheless, stock return predictions using the Shiller CAPE ratio have generally not performed well more recently. Beginning around 1985, average out-of-sample forecast errors of the predicted returns ten years ahead (i.e., 1995 and on) have been larger than if one had used the trailing historical long-run average. The rise in average forecast error has coincided with the secular rise in the CAPE ratio above its 1926-1984 average of 14.6. Indeed, the Shiller CAPE ratio has defied mean reversion since that time, having only once dropped below its long-run average. And realized U.S. stock returns over the past three decades have been robust, notwithstanding the global financial crisis.

Even with Shiller's CAPE ratio above a frothy 29 as of June 30th, 2017, industry experts do not agree on how much the U.S. stock market is "over-valued.[2]" Some focus on refining how the Shiller CAPE ratio is constructed. For instance, Siegel (2016) argues the secular rise in the CAPE ratio's trend is due to changes in accounting standards, and that NIPA (national income and product account) earnings should serve as the "E" in the CAPE ratio. Using a NIPA-based CAPE ratio, Siegel contends that U.S. stocks are not overvalued.

[2] See, for instance, Foster's (2017) CFA Institute blog at https://blogs.cfainstitute.org/investor/2017/06/13/shiller-vs-siegel-is-the-stock-market-overvalued/

The economic environment has also been cited as another factor in (justifying) elevated CAPE ratios. On a keynote panel at the 70th Annual CFA National Conference in May 2017, Professors Jeremy Siegel and Robert Shiller both cited low interest rates as a potential factor in the extended period of elevated CAPE ratios, although neither explicitly quantified the link between interest rates and future stock returns. This paper does just that.

Here we show that the primary issue is not with the Shiller CAPE ratio. Rather, it's with simple regressions that assume the CAPE ratio will revert mechanically to its fixed long-run average, regardless of the economic environment. We stipulate that the "fair-value" CAPE ratio (i.e., the value that the actual CAPE ratio should eventually revert to) varies over time, dependent on the state of the economy, as measured by real interest rates, expected inflation, and measures of financial volatility. In our framework, lower real bond yields imply lower real earnings yields and a higher "fair-value" CAPE ratio, all else equal. Real yields matter in our framework, not nominal yields *per se* as in the so-called Fed Model (Asness, 2003).

Our methodology is most similar to the pioneering work of Bogle (1991, 1995) and Bogle and Nolan (2015). The so-called Occam's razor model of Bogle and Nolan (2015) projects ten-year-ahead U.S. stock returns based on the current level of the dividend yield, the trailing 10-year-average in earnings growth, and a straight-lined reversion of the current P/E ratio to its trailing 30-year average. We attempt to maintain the elegant simplicity of Bogle and Nolan's approach, while refining and improving upon the assumption of—and economic rationale for—CAPE mean reversion. Both approaches tend to produce similar stock forecasts *unless* real bond yields differ from their long-run average at the time that the stock market forecast is made. That is certainly the case today; as of December 31st, 2016 our derived real ten-year Treasury yield was near 0%.

Our model's out-of-sample forecasts outperform the conventional approaches using Shiller's CAPE ratio, Siegel's CAPE ratio, and even the Occam's razor model of Bogle and Nolan (2015). Real-time forecast errors for ten-year-ahead U.S. stock returns are lower since 1960, and significantly so since 1985. Specifically, the average return forecast error of our two-step approach since 1985 is 4.1% (3.4%), versus 7.8% (6.2%) from a linear predictive regression using the Shiller (Siegel) CAPE ratio.

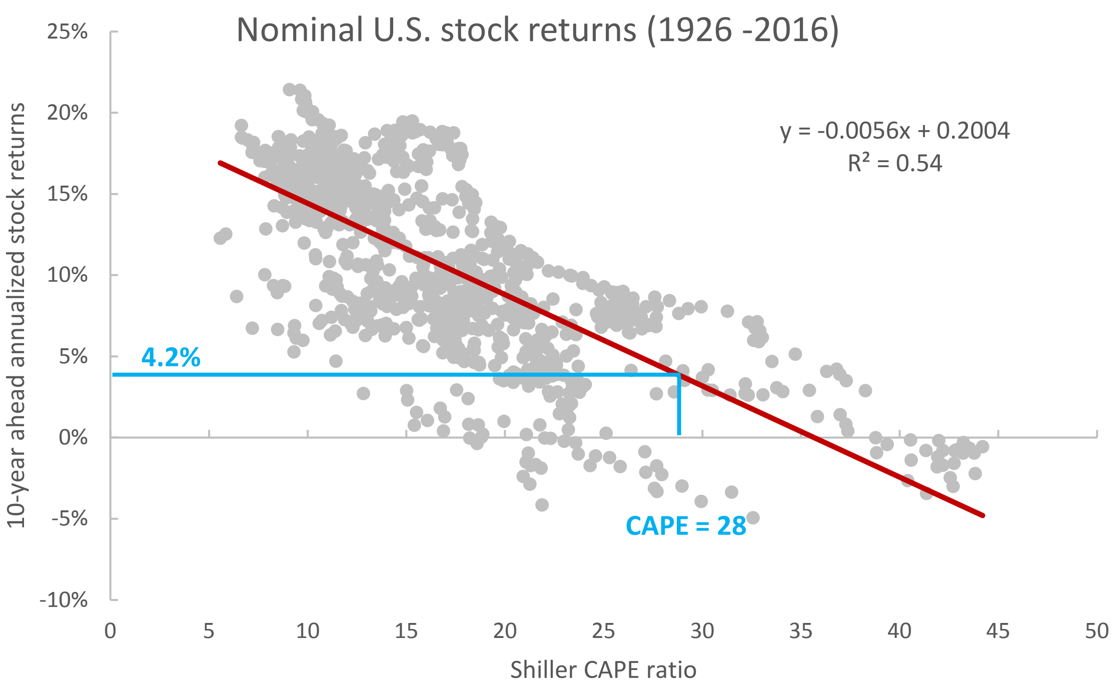
We conclude with a discussion of our model's low U.S. equity return projections over the next decade through 2026. In short, low real bond yields justify higher CAPE ratios today versus historical averages, yet they are very likely to prove insufficient in generating average stock returns over the next decade.

The conventional use of Shiller's CAPE ratio

Future long-run U.S. stock returns have tended to move inversely with the CAPE ratio over time, as illustrated in Figure 1 by the scatterplots for nominal returns (Panel A) and real returns (Panel B).[3] Financial analysts often reference such scatterplots in forecasting 10-year-ahead stock returns (real or nominal) by applying the downward-sloping red line in Figure 1 to the current value of the CAPE ratio.

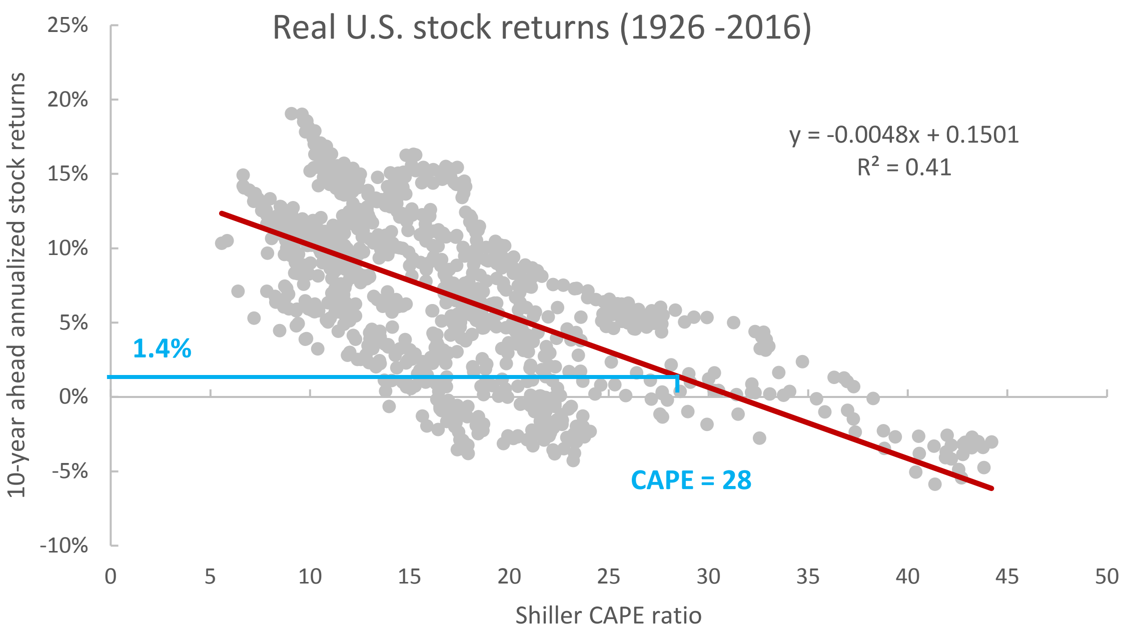
Figure 1: *CAPE ratio as a predictive regression*

Panel A: Nominal U.S. stock returns



[3] The challenges of predicting stock returns over shorter horizons are well documented in Campbell and Thompson (2008) and Goyal and Welch (2008). For a survey of the literature in predicting the equity risk premium, see Illmanen (2011), Damodaran (2012), and Davis et al (2012).

Panel B: Real (inflation-adjusted) U.S. stock returns



Sources: Authors' calculations based on data sources listed in the Data Appendix.

Statistically, this approach is equivalent to expressing monthly annualized 10-years-ahead stock returns as a linear function of the latest Shiller CAPE ratio via an OLS predictive regression:

$$(1) \qquad R_{t+120} = \alpha + \beta CAPE_t + \epsilon_t.$$

Figure 1 clearly shows that the CAPE ratio has explained a strikingly-high 54% of the time-series variation in 10-year-ahead nominal U.S. stock returns, as measured by equation (1)'s in-sample, or fitted, R^2, over the 1926-2016 period. Further enhancing the popularity of Shiller's CAPE ratio is that it peaked in 1929 and 1999 before noted stock market crashes.

CAPE ratio's forecast accuracy has deteriorated lately

Unfortunately, CAPE ratio's out-of-sample forecast accuracy has weakened since the mid-1980s versus its in-sample fit, as illustrated in Table 1. To be sure, the correlations between actual U.S. stock returns and those predicted ten years prior by the Shiller CAPE ratio have remained high in real time. Since 1960, the correlation has been 83%, and a remarkable 91% since 1985. But there is an important catch.

Table 1: *The CAPE ratio's predictive power out-of-sample*

Panel A: Nominal returns

	Out-of-sample forecasts made since 1960			Out-of-sample forecasts made since 1985		
Predictive variable	**Correlation of predicted returns with actual**	**Average forecast error (RMSE)**	**Model forecast error relative to error of using a naïve historical mean forecast**	**Correlation of predicted returns with actual**	**Average forecast error (RMSE)**	**Model forecast error relative to error of using a naïve historical mean forecast**
Historical average		5.8%			6.2%	
Shiller CAPE ratio	83%	5.5%*	LOWER	91%	7.8%***	HIGHER
Siegel CAPE ratio	67%	4.9%***	LOWER	90%	6.2%	SIMILAR

Panel B: Real returns

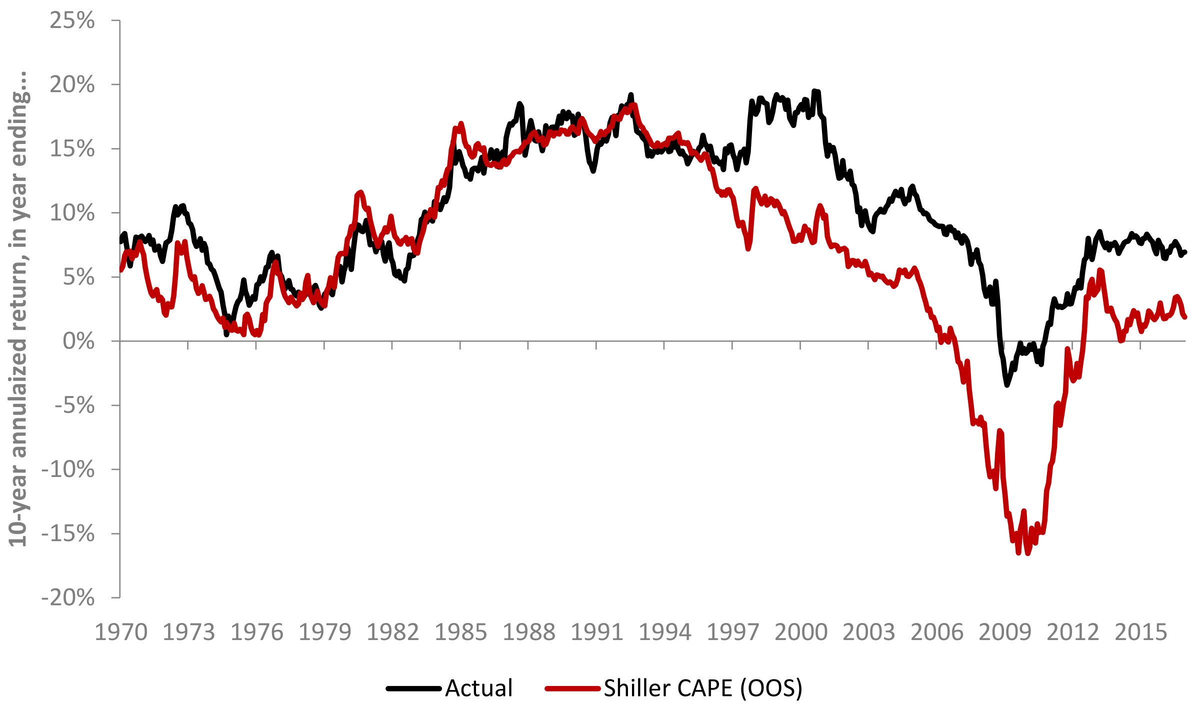
	Out-of-sample forecasts made since 1960			Out-of-sample forecasts made since 1985		
Predictive variable	**Correlation of predicted returns with actual**	**Average forecast error (RMSE)**	**Model forecast error relative to error of using a naïve historical mean forecast**	**Correlation of predicted returns with actual**	**Average forecast error (RMSE)**	**Model forecast error relative to error of using a naïve historical mean forecast**
Historical average		6.4%			5.7%	
Shiller CAPE ratio	56%	6.3%	SIMILAR	81%	7.8%***	HIGHER
Siegel CAPE ratio	36%	6.2%	SIMILAR	76%	5.8%	SIMILAR

Notes: The statistics shown are for 10-year annualized returns using the traditional predictive regression equation (1) with Shiller CAPE and Siegel CAPE. A "*" next to the RMSE refers to the significance of the Diebold-Mariano test (2002) of whether the forecast is statically better or worse than the historical mean. Significance level at 90%, 95% and 99% are denoted by one, two and three asterisks respectively.
Sources: Authors' calculations.

We must stress that correlation is *not* necessarily a reliable indicator of forecast accuracy.

A better measure of forecast accuracy is the average forecast error (i.e., RMSE) between the actual and predicted rolling 10-year-ahead returns. To stress this distinction, Figure 2 presents rolling actual versus the predicted 10-year annualized U.S. stock returns since 1960. While the CAPE-based predictions using equation (1) have been highly correlated with the actual future returns, the forecast error in absolute returns (the basis for RMSE) has generally grown over time. Beginning with long-run forecasts made in the mid-to-late 1980s, the Shiller CAPE ratio's projected stock returns have generally been too bearish, even when one includes the 1999 tech bubble. Put another way, Table 1 shows that investors would have been better served by using the historical average return as its baseline forecast of future stock returns over the past two decades rather than the scatterplots often employed in Figure 1.

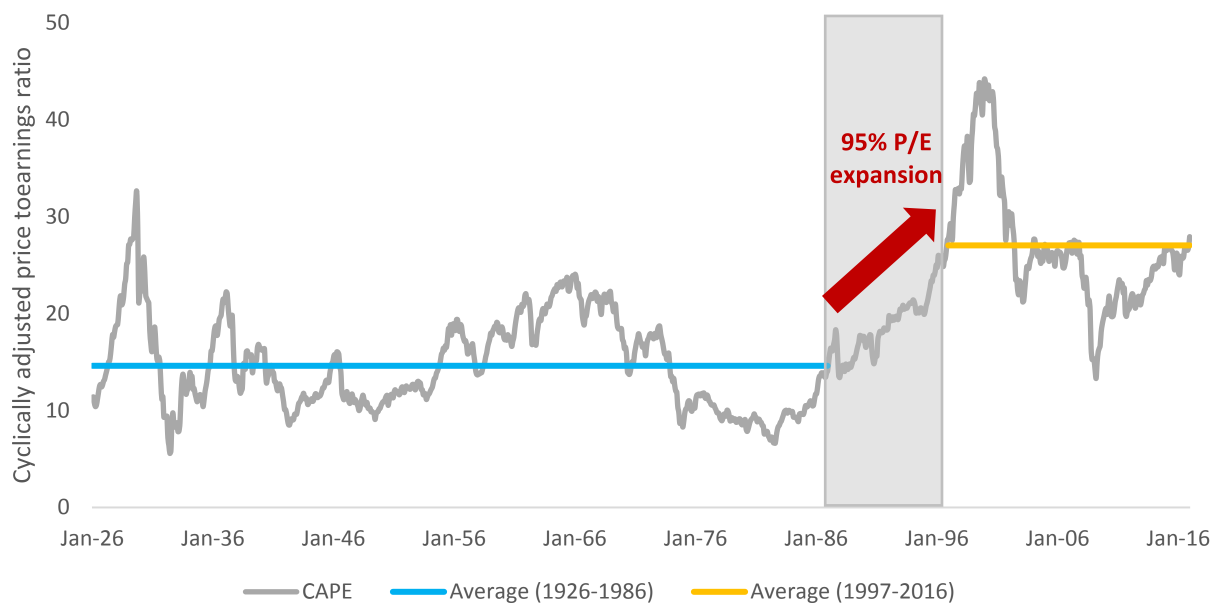
Figure 2: *The Shiller CAPE ratio's real-time forecasts since 1960*



Note: For the real-time analysis, the regression coefficients are determined recursively at a monthly frequency, starting with January 1926 - December 1959 data and re-estimating the regression coefficients every month thereafter. The gap between the two lines represents forecast error.
Source: Authors' calculations.

The basic explanation for the CAPE ratio's degradation in real-time forecasting power is that it has failed to converge meaningfully to its long-run historical mean. As illustrated in Figure 3, the CAPE ratio appears to have drifted secularly upwards since the late 1980s. Indeed, it has only dropped below its long-run 1926-2016 mean once since that time, albeit briefly, during the global financial crisis of 2009. There could be several reasons for this.

Figure 3: *Which mean will the CAPE ratio revert to?*



Source: Calculations based on the data obtained from Robert Shiller website, at aida.wss.yale.edu/~shiller/data.htm.

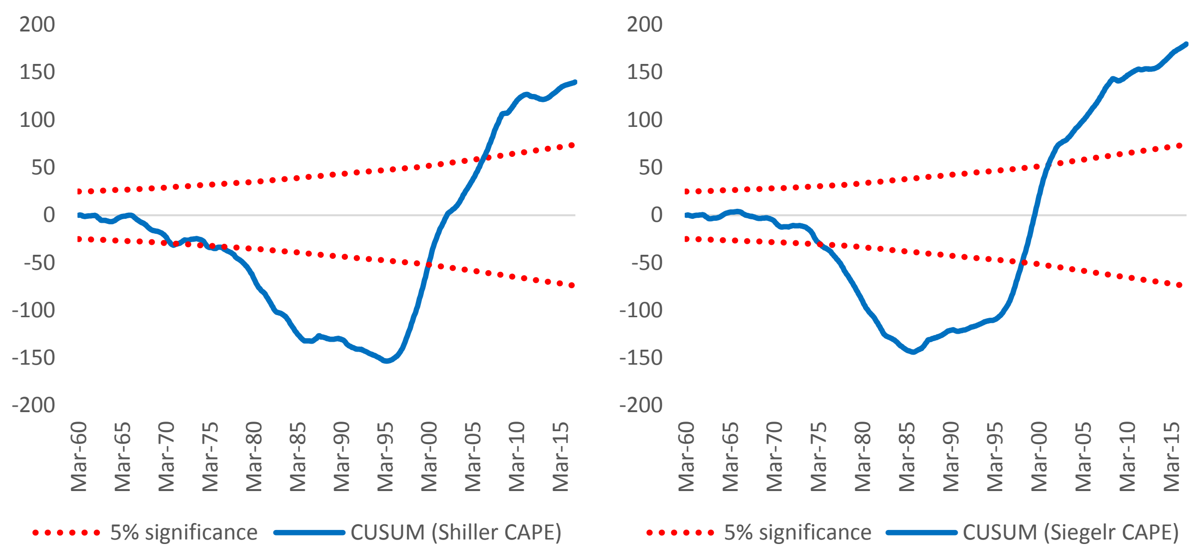
Siegel (2016) argues the rise in the CAPE's trend is primarily due to changes in accounting standards over time, and that NIPA (national income and product account) earnings should be substituted for GAAP earnings when applying the CAPE ratio. The bottom row of Table 1 shows that while real-time return projections since 1960 are marginally better using Siegel's CAPE ratio, its forecasts since 1985 have been statistically equivalent to the historical average, having roughly the same RMSE. Regardless of how we define or smooth earnings here, the real-time forecasting accuracy has been weaker than its in-sample fit. Changing the definition of "E" in the CAPE ratio has apparently not been a panacea for forecasting U.S. stock returns.

The issue is not with the CAPE ratio, but with CAPE regressions

The weak predictability of the CAPE ratio is less about the earnings used in its calculation, and is more a reflection of *model instability* (Goyal and Welch, 2008; Pettenuzzo and Timmermann, 2011). In other words, the estimated parameters in equation (1) for the average return that stocks revert to (dictated by the regression's conditional mean, or $\propto$), and the speed of the convergence (as governed by the regression's slope, or β) have not remained constant over time.

Figure 4: *Traditional CAPE regressions are unstable*

CUSUM tests using Shiller CAPE ratio **CUSUM tests using Siegel CAPE ratio**



Note: The CUSUM test (Brown, Durbin, and Evans 1975) for the 10-year-ahead stock return regression is based on the cumulative sum of the recursive residuals. The test finds parameter instability if the cumulative sum (blue line) extends beyond the area between the two dashed 5% significance (red) lines.
Source: Authors' calculations, based on data from Robert Shiller website, at aida.wss.yale.edu/~shiller/data.htm, U.S. Bureau of Labor Statistics and Federal Reserve Board.

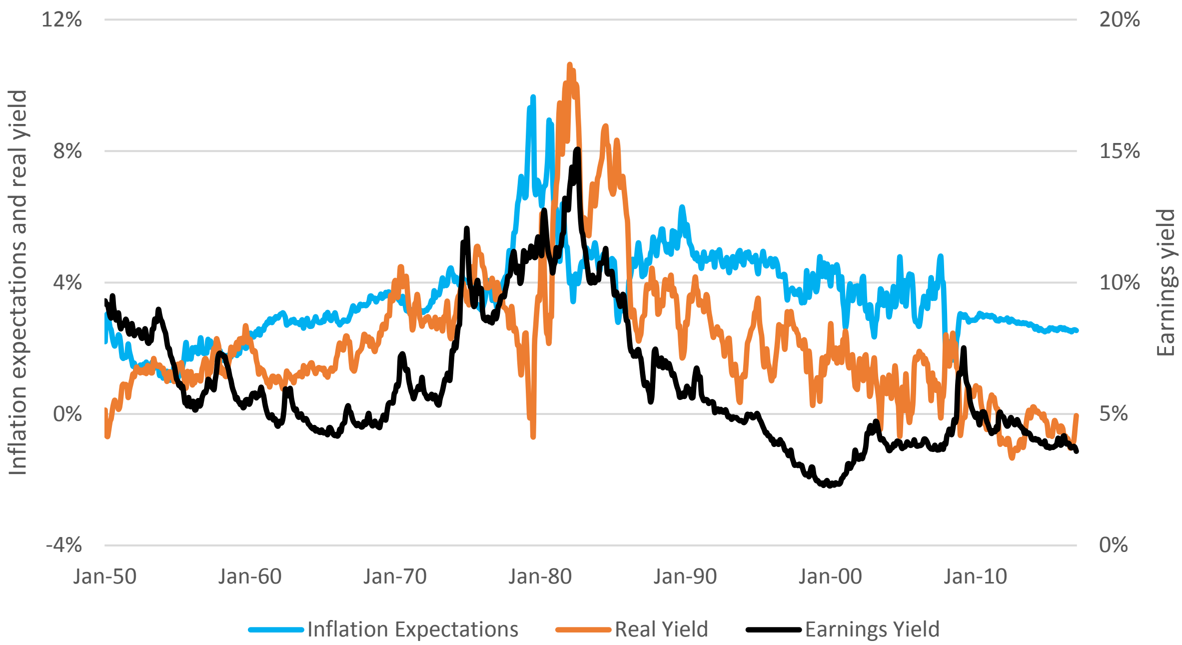
As evidence that traditional CAPE regressions suffer from model instability, Figure 4 presents the results of cumulative sum (or, CUSUM) tests of equation (1) using the Shiller and Siegel CAPE ratios, respectively. The lines of the CUSUM test signify parameter instability of conventional CAPE regressions, as the solid (blue) line breaches the 5% significance lines around 1985 or so. Figure 4 helps to explain the weak out-of-sample performance we document for both Shiller's and Siegel's CAPE ratios in Table 1 despite the high average correlation demonstrated in Figure 1.

Mean reversion is conditional on the economy

CAPE regression instability originates from at least two sources. The first is *estimation bias* that arises when persistent (or, "slow moving") variables such as the CAPE ratio are used to forecast long-run returns (Stambaugh, 1999). The second relates to standard CAPE regressions omitting the explicit relationship between the expected return on equity (i.e., the real earnings yield) and the expected real discount rate or cost of capital (i.e., real bond yields). If changes in long-term real interest rates influence the steady-state or "fair-value" CAPE ratio that stock returns should revert to, then the coefficients in a traditional CAPE regressions will suffer from instability whenever there are meaningful changes in the level of real bond yields.

This is precisely what we find. The solid lines in Figure 4 identify two major periods of instability for the traditional CAPE regression in equation (1): the late 1970s to mid-1990s and the post mid-2000s. This parameter instability implies that the CAPE ratio (and its inverse 1/CAPE, or real earnings yield) may not revert mechanically to a fixed, average mean. The low *real* interest rate environment may also explain why the CAPE ratio has not dropped below its long-run average of 16.9 since 1990, albeit for a brief time during the global financial crisis of 2009. The parameter instability in the CAPE regression appear to coincide with material shifts in average real bond yields, such as the high average real yields between the late 1970s and mid-1990s, and the secularly lower real yields before and after that period.

Figure 5: *The intuition: Higher real bond yields = high equity earnings yields*



Source: Authors' calculations. Please see Data Appendix.

An improved two-step approach using the CAPE ratio

Our hypothesis is simple: lower real bond yields should imply lower earnings yields and thus higher equilibrium or "fair-value" CAPE ratios, all else equal. The correlation between real bond and earnings yields in Figure 5 suggests that this is a reasonable approach.[4]

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[4] Inflation expectations appear less relevant and may explain why nominal bond yields (i.e., the Fed model) are not robust predictors of future stock returns. The results of our two-step model will illustrate this later.

Motivated by this insight, we propose a simple, two-step approach to forecast stock returns. While our model differs from the approach typically taken in equation (1), it can be estimated in real-time using standard software, it does not involve "look-ahead bias," and, for the U.S. stock market, it only requires the variables in the CAPE ratio data file conveniently provide by Professor Robert Shiller's website.

Our methodology is most similar to the original work of Bogle (1991, 1995) and Bogle and Nolan (2015). The so-called Occam's razor model of Bogle and Nolan (2015) projects ten-year-ahead U.S. stock returns based on the current level of the dividend yield, the trailing 10-year-average in earnings growth, and a straight-lined reversion of the current P/E ratio to its trailing 30-year average. We attempt to maintain the elegant simplicity of Bogle and Nolan's approach, while refining and improving upon the assumption of—and economic rationale for—CAPE mean reversion. Both approaches should produce similar stock forecasts *unless* real bond yields differ from their long-run average at the time that the stock market forecast is made.

Step 1: *A VAR model with earnings yields,* $1/CAPE$

Unlike traditional methods, we do *not* forecast returns directly, but rather forecast the inverse of the CAPE ratio itself. Specifically, we estimate a vector autoregressive (VAR) model with twelve monthly lags of the form:

$$(2)\ \mathrm{X_t} = \alpha + \beta_1 \mathrm{X_{t-1}} + \beta_2 \mathrm{X_{t-2}} + \cdots + \beta_{12} \mathrm{X_{t-12}} + \varepsilon_t,$$

where $\boldsymbol{X_t}$ is a vector of the five variables in the VAR model in logarithmic form, including:

- CAPE real earnings yield, or $1/CAPE$

- Real 10-year bond yields, or nominal Treasury yield less an estimated 10-year expected inflation rate (see Appendix)
- Year-over-year CPI inflation rate
- Realized S&P500 price volatility, over trailing 12 months, and
- Realized volatility of changes in our real bond yield series, over trailing 12 months.[5]

The motivation of including these five VAR variables derives from Asness (2003), who finds that earnings yield rises when bond yields rise, when stock volatility rises, and when bond market volatility falls. Note that we lag the "E" in the CAPE ratio by six months and the CPI data two months to account for real-time data availability at any month end.

Step 2: *Impute stock returns from the CAPE earnings yield forecasts*

Rather than estimating equation (1), we calculate future returns directly based on their three components, thereby reducing estimation bias. We adapt the framework of Bogle and Nolan (2015) and Ferreira and Santa-Clara (2011) in imputing monthly stock returns by their "sum of parts" identity:

$$(3)\ r_{t+1} \equiv \%\Delta PE_{t+1} + \%\Delta E_{t+1} + DP_{t+1}$$

where %ΔPE is the percentage change in P/E ratio, %ΔE is earnings growth, and DP is the dividend yield. The VAR model's forecast for the earnings yield provides us the percentage changes in CAPE ratios, $\%\Delta PE_{t+1}$, for imputing stock returns directly by the "sum of parts"

[5] The motivation of including these five VAR variables derives from Asness (2003), who finds that earnings yield rises when bond yields rise, when stock volatility rises, and when bond market volatility falls. Arnott, Chaves, and Chow (2015) find that both real yields and inflation expectations are positively related to the earnings yield on U.S. stocks. It remains unclear why inflation expectations – a component of nominal bond yields – should influence earnings yields since stocks are a long-run inflation hedge (Illmanen, 2011, ch. 8). Importantly, this so-called "inflation illusion" effect is weaker in our VAR model than the effect from real bond yields given the joint dynamics of our VAR model, which we discuss below.

equation (3). For simplicity, we assume that earnings growth is constant and equal to its long-term average, while the dividend yield is the product of the earnings yield times the payout ratio.[6] As a result, only earnings yield $(1/CAPE)$ has to be forecasted via regression in order to predict stock returns at a given horizon. At any point in time, the VAR can forecast out for ten years the CAPE earnings yield and, via step 2, derive an expected future 10-year-ahead return on U.S. stocks. Table 2 summarizes the similarities and differences between our approach compared to (a) traditional Shiller CAPE regressions / scatterplots, and (b) the Bogle Occam's razor model.

Table 2: *Comparison of different stock forecasting approaches*

	Ingredients in the stock-return forecasts		
Stock return component	**Traditional Shiller CAPE ratio regression**	**Bogle Occam's razor model**	**This paper's two-step approach**
Dividend yield	Swept into OLS alpha / intercept coefficient	Actual value at beginning of period	Derived from forecasted earnings yield (below) times the beginning of period payout ratio
Earnings growth	Swept into OLS alpha / intercept coefficient	Uses trailing 10-year earnings growth	Uses trailing long-run historical average earnings growth
CAPE ratio "mean reversion" process	Estimated by "beta" of the regression; not conditional on any other variables	Linear proration over ten years between last available PE ratio and trailing 30-year average; provides "speculative return"; not conditional on any other variables	Forecased earnings yield from 5-variable VAR model that also includes real bond yields, inflation, real bond volatility and equity volatility

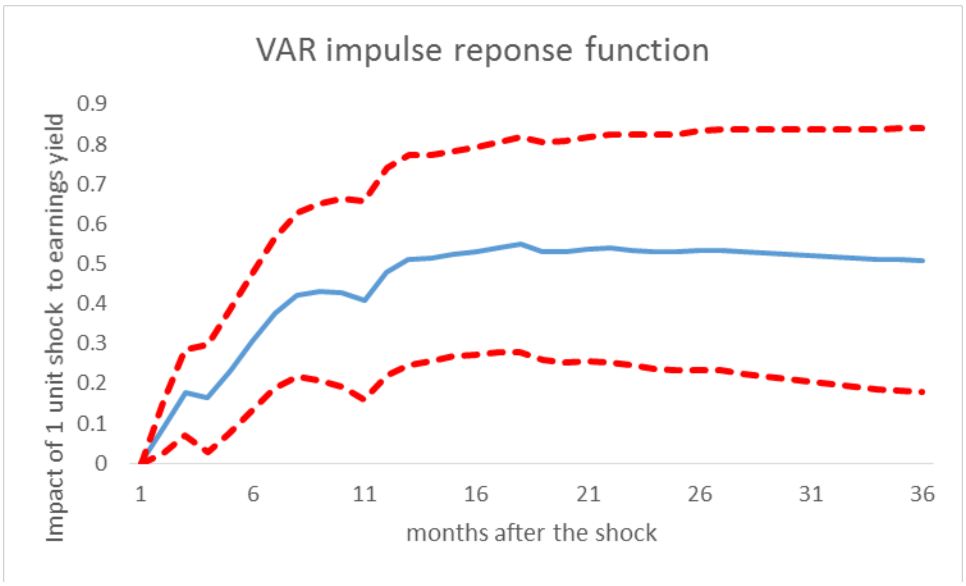
Sources: Authors' calculations.

The potential benefit of our approach is that the "fair-value" CAPE ratio—which the actual CAPE ratio should revert back to—is permitted to vary over time conditional on the movements

[6] The benefit of our "sum of parts" approach is that it should mitigate so-called Stambaugh (1999) bias that can plague predictive regressions with persistent regressors like CAPE ratios that involve overlapping data (Nelson and Kim, 1993). In results unreported here but available upon request, including changes in earnings growth in the VAR does not materially alter the results. Consistent with Cochrane (2008), changes in earnings yields help predict future stock returns, not earnings growth.

in these other fundamental variables.[7] It is this "fair-value" CAPE that should be the relevant benchmark for forecasting the equity risk premium, not the CAPE long-run average.[8] Put another way, if actual CAPE ratios revert back to our fair-value CAPE ratio and not CAPE's historical average, then our two-step model should produce more reliable stock return forecasts than traditional Shiller CAPE regressions.

Figure 6: *Shocks to real bond yields lead to higher CAPE earnings yields*



Sources: Authors' calculations. Dotted red lines reflect standard error bands.

The VAR model dynamics for the earnings yield are intuitive. Figure 6 traces the impulse response function of the earnings yield (1/CAPE) to shocks to real bond yields. Movements in earnings yields are jointly determined by changes in real bond yields, as both are measures of

[7] To predict future stock returns, we need not forecast the other five VAR factors accurately in as much as account for their inter-dynamics in affecting earnings yields through time.
[8] The notion of a variable's unobserved "fair value" is common in macroeconomics. Examples include the full-employment concept of NAIRU, a currency's PPP, and the natural rate of interest, or "R-star."

expected future economic growth and monetary policy.[9] The intuition for the positive correlation between real bond yields and stock earnings yield is simple; lower expected economic growth implies lower real bond yields, which implies lower earnings yield on stocks, and thus a higher "fair value" CAPE ratio, all else equal. The disinflationary period and bond-bull market of the 1980s coincided with rising stock valuations. As real interest rates fell below their historical averages in the 1990s and 2000s, equity earnings yield remained below their own average levels, too.

Comparing real-time forecasts: An illustration

Figure 7 compares the projections for the earnings yield (1/CAPE) from two models: (a) that which is implied by a traditional Shiller CAPE regression[10], and (2) our VAR model. For convenience, we re-express the earnings yield as the CAPE ratio. We choose December, 1999, when the CAPE was above 40, to illustrate relative forecast performance.

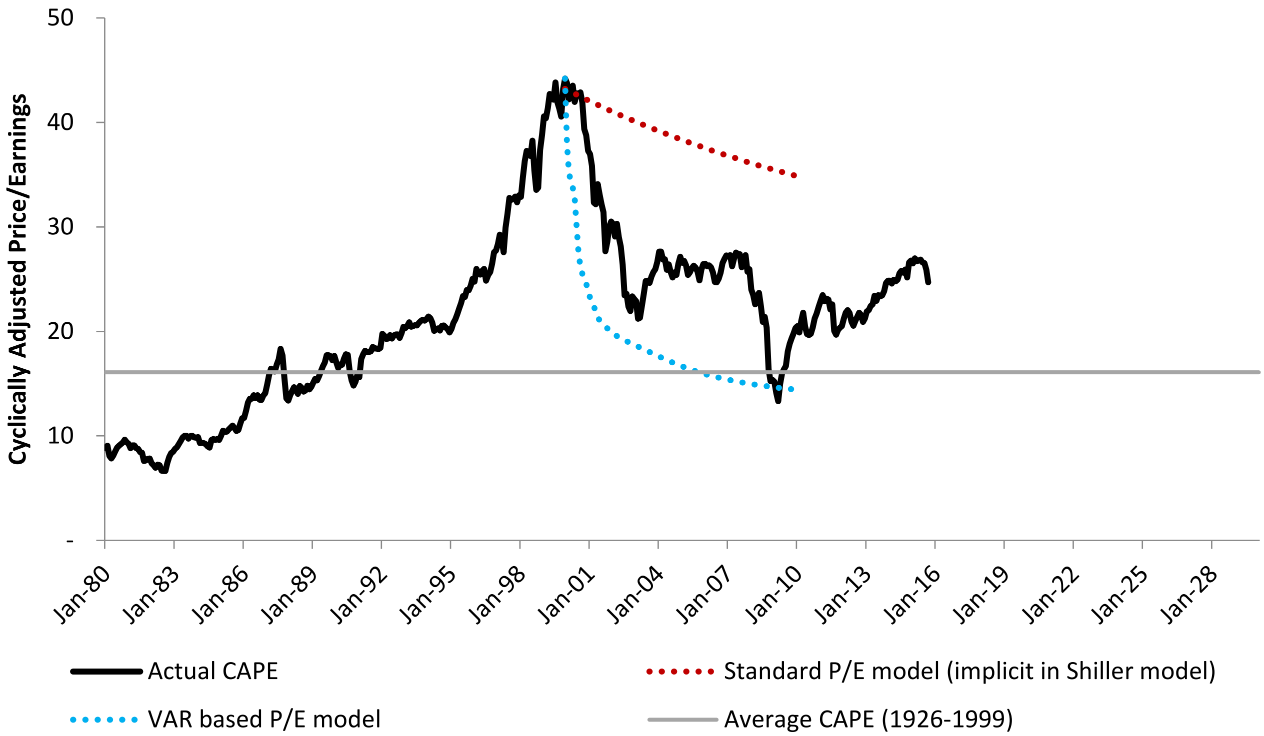
Following the dot-com bust period after 1999, the VAR-based CAPE projections anticipate subsequent CAPE trends more accurately than even equation (1). This is because earnings yields are not assumed to converge unconditionally to their long-run average as typical Shiller CAPE regressions do, but rather are a function of the current state of other variables in the model. Rising real bond yields—combined with the CAPE ratio above its fair value—leads to a sharper

[9] Historically, earnings yield and real bond yields have tended to move in tandem. We also know that "breaks" in real yields and inflation expectations occurred during the early 1950s (the end of the Treasury-Fed accord that pegged interest rates after WWII), the mid-1970s (the OPEC oil shock) and the early 1980s (when Volcker and the Fed tamped down higher inflation) given changes in macroeconomic regimes. In results unreported here, we link the structural breaks in Shiller CAPE regressions to breaks in real interest rates and other financial conditions which, when controlled for, lead to improved model stability. That is, the VAR equation for the earnings yield does not suffer from structural breaks in either its conditional mean (alpha) or the speed of mean reversion (beta) when the other variables of our VAR model are included.

[10] It can be shown that any predictive regression is equivalent to a single-period stock return equation plus an AR(1) or first-order regressive process in the predictor.

correction in earnings yields in our VAR model, and hence more accurate future stock market returns. This case study underscores why "conditioning" mean-reversion in CAPE ratios on real bond yields can improve long-run return forecasts.

Figure 7: *CAPE real-time forecasts, VAR model vs implied by traditional Shiller regressions*



Note: Vanguard's model uses a VAR based P/E model.
Sources: Vanguard calculations, based on Robert Shiller's website, at aida.wss.yale.edu/~shiller/data.htm

Comparing forecasting performance

Table 3 compares the predictability of our two-step approach to the Shiller and Siegel CAPE ratios by running out-of-sample forecasts for the U.S. stock market for same two periods as before: 1960 to 2016 and 1985 to 2016. Over this period, our approach forecasts 10-year-ahead stock returns more accurately (in real-time) for the United States. In real time, the two-step model's RMSE is lower and statistically different from the historical average for both Shiller and

Siegel valuation metrics. Since 1985, the average forecast error (i.e., RMSE) using the two-step VAR model has been 4.1% compared to 7.9% for the Shiller CAPE ratio using equation (1), a reduction of more than 40%.

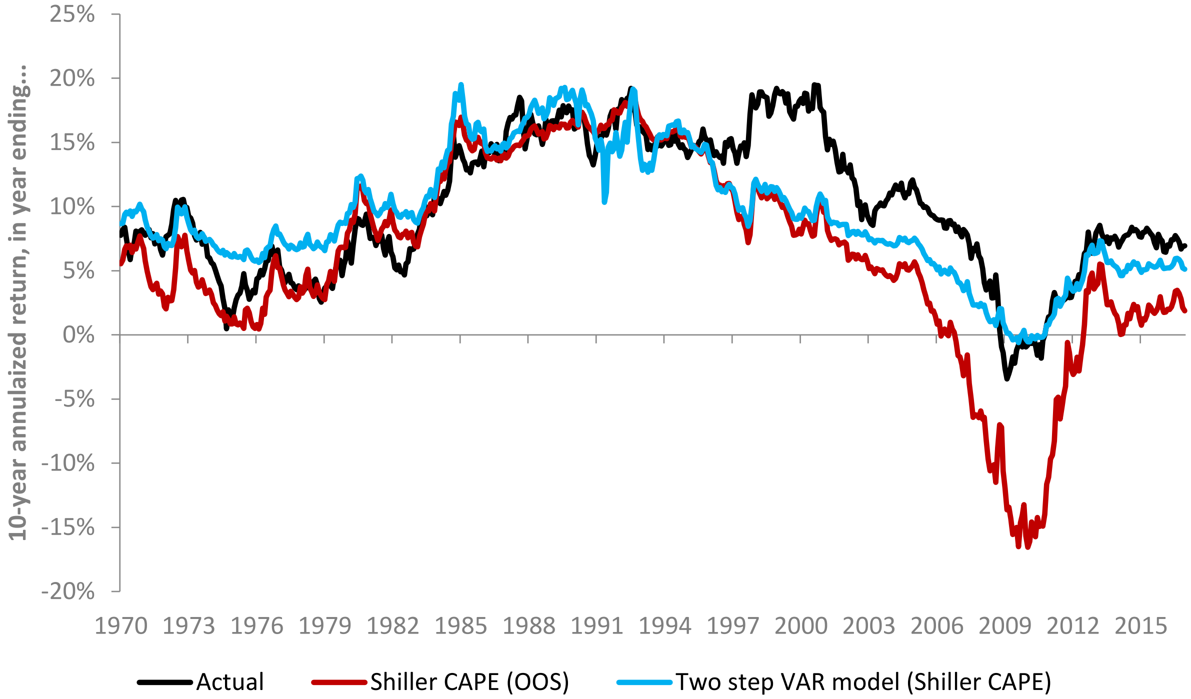
Table 3: Comparison of real-time predictive power, nominal U.S. stock returns

	Out-of-sample forecasts made since 1960			Out-of-sample forecasts made since 1985		
Predictive variable	**Correlation of predicted returns with actual**	**Average forecast error (RMSE)**	**Model forecast error relative to error of using a naïve historical mean forecast**	**Correlation of predicted returns with actual**	**Average forecast error (RMSE)**	**Model forecast error relative to error of using a naïve historical mean forecast**
Historical average		5.8%			6.2%	
Shiller CAPE ratio	83%	5.5%*	LOWER	91%	7.8%***	HIGHER
Siegel CAPE ratio	67%	4.9%***	LOWER	90%	6.2%	SIMILAR
Bogle Occam's razor	73%	4.7%***	LOWER	73%	5.9%	SIMILAR
Two step VAR model, Shiller CAPE	81%	3.2%***	LOWER	90%	4.1%***	LOWER
Two step VAR model, Siegel CAPE	67%	4.5%***	LOWER	90%	3.4%***	LOWER

Notes: RMSE stands for root mean square error. The statistics shown are for 10-year annualized returns using the models described. A "*" next to the RMSE refers to the significance of the Diebold-Mariano test (2002) of whether the forecast is statically better or worse than the historical mean. Significance level at 90%, 95% and 99% are denoted by one, two and three stars respectively.
Source: Authors' calculations based on the data sources listed in the appendix.

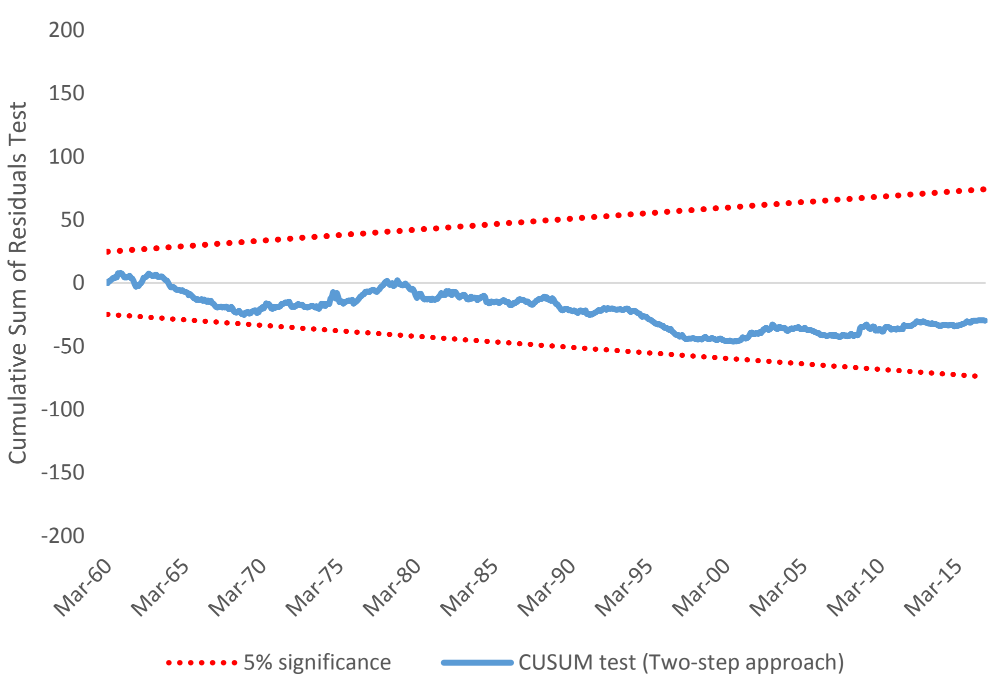
Figure 8 shows the actual real-time forecast of our two-step model for U.S. stocks. Our fair-value CAPE approach tracks the actual rolling 10-year-ahead U.S. stock returns fairly well, declining throughout the 2000s and anticipating a strong rebound immediately following the global financial crisis in 2009. Traditional CAPE regressions are also highly correlated with future returns, yet they consistently project lower 10-year-ahead stock returns than what has been actually realized by investors over our sample period. Figure 9 shows that in contrast to the traditional CAPE models, our two-step approach exhibits better parameter stability in the out-of-sample period.

Figure 8: *Two-step "fair-value" CAPE model—Reasonable out-of-sample performance*



Note: For the real-time analysis, the regression coefficients are determined recursively at a monthly frequency, starting with January 1926 - December 1959 data and re-estimating the regression coefficients every month thereafter.
Source: Authors' calculations.

Figure 9: *Two-step "fair-value" CAPE model appears more stable*



Conclusion

Valuation metrics such as price-earnings ratios are widely followed by the investment community because they are believed to predict future long-term stock returns. Arguably the most popular is Robert Shiller's cyclically-adjusted P/E ratio (or CAPE) which is currently above its long-run average. However, the out-of-sample forecast accuracy of stock forecasts produced by CAPE ratios has become increasingly poor. In this paper we have shown why and offer a solution to offer a more robust approach to produce long-run stock return forecasts.

The problem is not with the CAPE ratio, but with CAPE regressions. We show that a common industry approach of forecasting long-run stock returns can produce large errors in forecasted returns due to both estimation bias and its strict assumption that the CAPE ratio will revert over time to its long-run (and constant) mean. Although far from perfect, our model's out-of-sample forecasts for ten-year-ahead U.S. stock returns since 1960 are roughly 40-50% more accurate than conventional methods. Real-time forecast differences in 10-year-ahead stock returns are statistically significant, and have grown to exceed three percentage points after 1985 given the secular decline in real bond yields. In our model, lower real bond yields imply higher equilibrium CAPE ratios. This framework would appear to explain *both* elevated CAPE ratios and robust stock returns over the past two decades. Future research could involve testing our approach to non-U.S. markets with long-spanning data, or even sectors of the U.S. equity market.

Overall, we encourage investment professionals to adopt our straightforward framework when forecasting stock returns for strategic asset allocation. Our fair-value approach can be estimated in real-time using standard software, it does not involve "look-ahead bias," and, for the U.S.

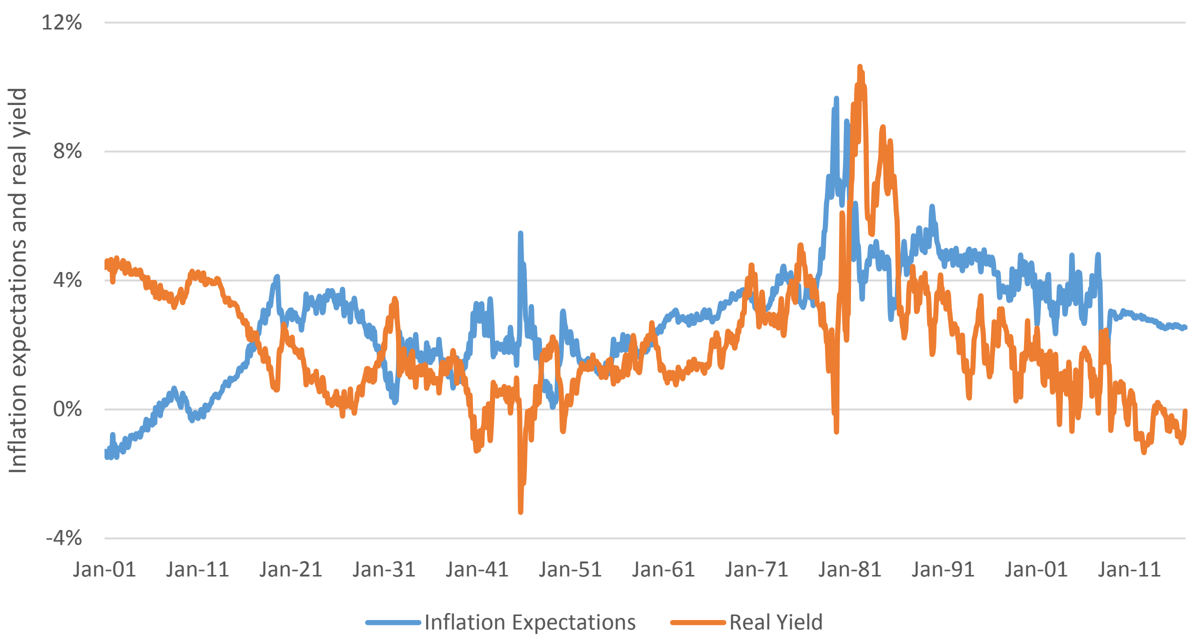
stock market, it only requires the variables in the CAPE ratio data file conveniently provide by Professor Robert Shiller's website. As of June 2017, our model projects a guarded, lower-than-historical return on U.S. stocks of approximately 4.9% over the coming decade. This muted forecast for U.S. stock returns is not simply because the CAPE ratio is above its long-run mean.

Data appendix

All of the data used in this paper were obtained from Professor Robert Shiller's website, at aida.wss.yale.edu/~shiller/data.htm. Real bond yields reflect the nominal 10-year U.S. Treasury yield less an estimate of 10-year-ahead inflation expectations. A consistently-defined and long-running series on U.S. inflation expectations since the 1920s does not exist.

Our synthetic inflation expectations series was derived so that an investor could have replicated them at the time our stock forecasts were made. Specifically, we defined inflation expectations as the average of the predicted CPI inflation rate over the next 10 years generated from an autoregressive model at any month in time. The AR model included 12 monthly lags in annualized CPI inflation rates and was estimated using a 30-year rolling window. The synthetic time series for our expected 10-year inflation rate is presented below.

Figure A1: *Inflation expectations and real yields*



Note: The model is an AR(12) model on monthly inflation with a 30 year rolling window. Initial estimation period is 01/1871 through 12/1900 after which monthly inflation is forecasted out for 10 years and annualized over 10 years to determine the inflation expectation in 01/1901. The estimation window is rolled forward estimate the inflation expectation series.
Source: Authors' calculations.

References

Arnott, Robert D., Denis B. Chaves and Tzee-man Chow. 2015. "King of the Mountain: Shiller P/E and Macroeconomic Conditions." Available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2640327, accessed in April, 2017.

Asness, Clifford S. 2000. "Stocks versus Bonds: Explaining the Equity Risk Premium." *Financial Analysts Journal*, vol. 56, no. 2: 96–113.

——. 2003. "Fight the Fed Model." *Journal of Portfolio Management*, vol. 30, no. 1: 11–24.

Bogle, John C. 1991. "Investing in the 1990s: Occam's Razor Revisited." *Journal of Portfolio Management*, vol. 18, no. 1: 88–91.

——. 1995. "The 1990s at the Halfway Mark." *Journal of Portfolio Management*, vol. 21, no. 4: 21-31.

Bogle, John C. and Michael W. Nolan, Jr. 2015. "Occam's Razor Redux: Establishing Reasonable Expectations for Financial Market Returns." *Journal of Portfolio Management*, vol. 42, no. 1 (Fall).

Brown, Robert L., James Durbin, and James M. Evans. 1975. "Techniques for Testing the Constancy of Regression Relationships over Time." *Journal of the Royal Statistical Society, Series B (Methodological)*, vol. 37, no. 2: 149-192.

Campbell, John Y. and Robert J. Shiller. 1988. "Stock Prices, Earnings, and Expected Dividends." *Journal of Finance*, vol. 43, no. 3: 661–676.

——. 1998. "Valuation Ratios and the Long-Run Stock Market Outlook." *Journal of Portfolio Management*, vol. 24, no. 2: 11–26.

Campbell, John Y. and Tuomo Vuolteenaho. 2004. "Inflation Illusion and Stock Prices." *American Economic Review*, vol. 24, no. 2: 11–26.

Campbell, John Y. and Samuel B. Thompson. 2008. "Predicting Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average? *Review of Financial Studies*, vol. 21, no. 4: 1509-1531.

Cochrane, John H. 2008. "The Dog that Did Not Bark: A Defense of Return Predictability." *Review of Financial Studies*, vol. 21, no. 4: 1533-1575.

——. 2011. "Discount Rates." *The Journal of Finance*, vol. 66:, no. 4: 1047-1108.

Damodaran, Aswath. 2012. *Equity Risk Premiums (ERP): Determinants, Estimation and Implications.* Available at http://ssrn.com/abstract=2027211, accessed in January, 2017.

Davis, Joseph H., Roger Aliaga-Diaz, and Charles J. Thomas. 2012. "Forecasting Stock Returns: What Signals Matter, and What Do They Say Now?" *Vanguard White Paper*.

Diebold, Francis X., and Robert S. Mariano. 2002. "Comparing Predictive Accuracy." *Journal of Business and Economic Statistics*, vol. 20, no. 1: 134-144.

Ferreira, Miguel, A. and Pedro Santa-Clara. 2011. "Forecasting Stock Market Returns: The Sum of the Parts is More than the Whole." *Journal of Financial Economics*, vol. 100, no. 3: 514–537.

Goyal, Amit, and Ivo Welch. 2008. "A Comprehensive Look at the Empirical Performance of Equity Premium Prediction." *Review of Financial Studies,* vol. 21, no. 4: 1455-1508.

Ilmanen, Antti. 2011. *Expected Returns: An Investor's Guide to Harvesting Market Rewards*. West Sussex, United Kingdom: John Wiley & Sons.

Nelson, Charles R., and Myung J. Kim. 1993. "Predictable Stock Returns: The Role of Small Sample Bias." *Journal of Finance*, vol. 48, no. 2: 641-661.

Shiller, Robert. 2000. *Irrational Exuberance*. Princeton, NJ: Princeton University Press.

Siegel, Jeremy J. 2016. "The Shiller CAPE Ratio: A New Look." *Financial Analysts Journal,* vol. 72, no. 3: 41–50.

Stambaugh, Robert F. 1999. "Predictive Regressions." *Journal of Financial Econometrics*, vol. 54: no. 3: 375- 421.